VIII. Appendix: Obtaining carbon content

The economy is a web of exchanges. The input that a supplier delivers to its customer may have required, at the end of a long chain, the output of this same customer for its manufacturing. This is the problem of circularity mentioned above. But it does not impede the calculation of the carbon content of all goods and services.

To show this, we simplify the country's production system to the extreme. It is composed of three sectors of activity (or firms) and three goods: a sector ‘e’ that supplies primary carbon exclusively from imports; a sector ‘i’ that manufactures an industrial good and sells it to itself, to the service sector and to households for their consumption; and a sector ‘s’ that provides a service to the industrial company, to itself and to households.

Considering for instance the carbon content of the service company, we need to know, in addition to the direct energy consumption of one unit of the service product (scope 1), the intermediate consumption of industrial goods and services by this company, since these inputs themselves have a direct carbon content. For that matter, the whole set of ramifications must be followed step by step upstream, here limited to two companies.

The table below represents the productive structure of the economy.

	Industrial Product	Service Product	Purchase of fossil energy (scope 1)
Industrial Firms (i)	0,6	0,5	10
Service Firms (s)	0,2	0,2	6

The first line of the table represents the consumption of the industrial firm in the two products, industrial and service. These are physical quantities: for example, 0.6 is the quantity of the industrial good (i.e., number of machines) that it takes to make one unit of the industrial good; 0.5 is the quantity of service (i.e., in units of time) for one unit of the industrial good. Such a table is similar in simplified form to the input-output table or Leontief table that is used in national accounts worldwide. It is with the help of this tool, extended at international level, that INSEE (2022) proceeds in the note in reference to calculate the carbon footprint for France.

The last column shows the direct energy demand (scope 1) of the two firms for their unit production. Here, $d_i = 10$ et $d_s = 6$, with the index *i* or *s* depending on the company, the unit this time being the ton of CO_2.

There is now enough information to know immediately the carbon content directly and indirectly contained in each unit of the two goods, and, by multiplying by the quantities produced, the total carbon content of each of the goods sold by the two companies.

Such carbon content or emission factor of the product sold by the industrial company is denoted by q_i. It adds up $d_i = 10$, its direct consumption, plus 0.6 times the carbon

content of the industrial product, unknown at this stage, plus 0.5 times the carbon content of the service product, also unknown. The same goes for q_s, the carbon content of the service produced by the service company.

$q_i$and q_s obey the following accounting equations:

$$\begin{cases} q_i = d_i(=10) + 0{,}6q_i + 0{,}5q_s \\ q_s = d_s(=6) + 0{,}2q_i + 0{,}2q_s \end{cases}$$

These two equations determine the direct and indirect carbon contents of a unit of the goods produced by the two firms. It gives $q_i = 50$ and $q_s = 20$.

Generally speaking, in an *n*-good economy, the carbon accounting of firm or sector *i* reads:

$$q_i = d_i + a_{1,i}q_1 + a_{2,i}q_2 + \cdots + a_{j,i}q_j + \cdots + +a_{n,i}q_n.$$

Calling *A* the n x n matrix of coefficients $[a_{j,i}]$, all of which are positive or zero, we more efficiently write:

$q = d + Aq$, which gives the solution for the carbon contents of each good in the economy, with the vectors *q* and *d* figuring the *n* total (*q*) and direct (*d*) carbon contents. It gives:

$q = (I - A)^{-1}d,$, where *I* is the unit n-matrix.

We show, under fairly large conditions[1], that there exists a solution with q > d. We thus have the announced result #1:

1. *The carbon contents of all goods and services are in theory immediately computable.*

We must now show that it is sufficient to have the partial information of the direct or scope 1 carbon contents (the vector *d* in the formalization), to obtain the same result. In short that the iterative process is convergent.

Let us assume that these direct contents are passed on downstream from the sellers to their customers. At the end of this first step, each firm then gets not only its direct consumption as an information, but also the one of the suppliers who immediately precede it. Formally, using the input-output table *A*, firms declare *d*, the direct content, plus *Ad*, the carbon contents of the goods of tier 1 suppliers.

[1] The economy must be "productive", i.e. intermediate consumption is less than the gross output of each branch. Technically, the matrix A, with positive or zero coefficients, must have eigenvalues less than 1. Note the similarity of the problem with that of calculating the labor content of goods in Ricardo's or Marx's labor-value theory, which shows in passing, according to a result due to Okishio and Morishima, and anticipated by Sraffa, that we can have labor values as well as carbon values or any other good, under the condition of a productive economy.

By the same reasoning, firms will add, at the next cycle of trade, the direct consumption of the suppliers of rank 2, i.e., in total: $d + Ad + A^2d$. We would thus approach, after a certain number of cycles, the true carbon contents, knowing that:

$d + Ad + A^2d + \cdots + A^td$ tends to $(I - A)^{-1}d = q$ as the number of cycles *t* increases.

Thus, we have shown result #2 announced above:

2. *The iterative process is convergent, even if firms initially transmit only the direct carbon content of their inputs.*

However, as explained above, the process converges much faster if each firm that has the means to do so includes in its invoices, in addition to the direct content, an expert estimate of the indirect cost. Gradually, companies will use the costs declared by their suppliers rather than their estimates.